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FORCED VIBRATIONS OF AN ELASTIC CIRCULAR CYLINDRICAL BODY OF FINITE LENGTH SUBMERGED IN AN ACOUSTIC FLUID

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Abstract—A method is presented for determining the fluid pressure and velocity fields produced by the forced vibrations of an elastic circular cylindrical body of finite length submerged in an acoustic fluid. A potential theory approach is utilized in which the three potentials associated with the elastic body and the one potential associated with the acoustic fluid are interpreted in terms of a distribution of four sets of sources of unknown strength over the fluid-elastic body interface. For a given excitation, compatibility conditions on the stresses and velocities on the fluid–body interface lead to the evaluation of the source strengths and the subsequent determination of the pressure and velocity fields in the fluid.

NOTATION*

r, θ, z	cylindrical coordinates locating source points
$\bar{r}, \alpha, \bar{z}$	cylindrical coordinates locating field point
u, v, w	longitudinal, tangential and radial displacements of the cylindrical shell
a	radius of cylinder
c_1	velocity of dilational waves in elastic cylinder
c_2	velocity of shear waves in elastic cylinder
c	velocity of sound in water
G, H, L, C	lumped source strength coefficients per unit of circumferential length
i	source band index
j	field point index
$k, k_{1,2,3}$; $\bar{k}, \bar{k}_{1,2,3}$	coefficient ω/c evaluated for $c = c_1, c_2, c$ as indicated in text; $\bar{k}$ is non-dimensional form $\omega a/c$
L	length of cylinder
N	number of bands into which cylinder is divided
n	number of circumferential waves in the cylinder displacements
n, n'	normal to a surface
p	pressure in fluid
P	expansion coefficient of externally applied normal tractions to cylinder
R	distance between field point and source point
R_0	distance between field point and origin, Fig. 5
t	time
$\alpha_{nji}, \beta_{nji}, \gamma_{nji}$	integral coefficients of source strengths
Δ	band width: see Fig. 2
λ, μ	elasticity constants of cylinder
Φ	potential function associated with velocities of acoustic fluid
φ, ψ, η	potential functions associated with displacements of elastic cylinder
ψ	angle between field and source points
ρ_f	mass density of fluid
$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$; $\sigma_{r\theta}, \sigma_{rz}, \sigma_{z\theta}$	cylinder stresses

* Additional symbols are defined as they occur in the text.

ω frequency of vibration
ζ slope of R_0: see Fig. 5

Note: r or z used as a subscript for the potential functions denotes differentiation with respect to the particular variable used.
r or z appearing as a superscript for the coefficients α_{nji}, β_{nji}, γ_{nji} denotes differentiation with respect to the particular variable used. Dots indicate differentiation with respect to time.

1. INTRODUCTION

THIS paper presents a study on the development of methods for treating the forced vibrations of elastic bodies of revolution submerged in an acoustic fluid. Specifically, the pressure and velocity fields which are produced in the fluid by the harmonic excitation of a solid elastic circular cylindrical body of finite length (Fig. 1), are evaluated.

A potential theory approach is used in which the stresses and velocities in an elastic cylinder of finite length are expressed in terms of three potential functions, each of which

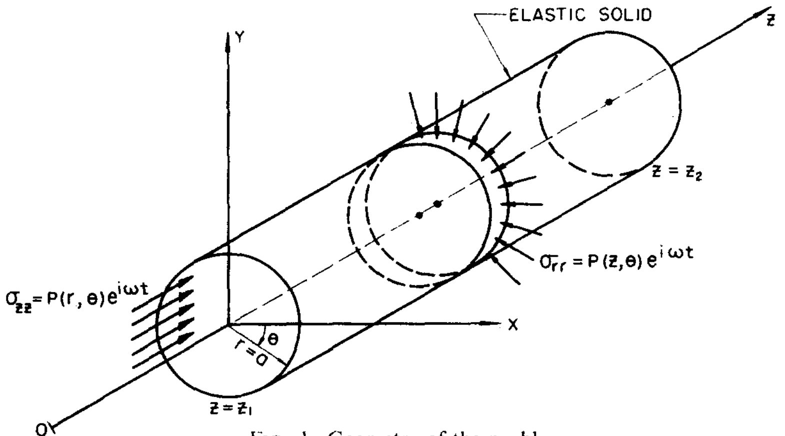

FIG. 1. Geometry of the problem.

satisfies the wave equation. Similarly, the corresponding quantities in the fluid are expressed in terms of a single fluid potential which also satisfies the wave equation. Each of the four potential functions can be considered to be caused by a group of sources of unknown strength which are distributed over the common boundaries of the elastic body and the fluid surface. Conditions on the stresses and velocities at the elastic body–acoustic fluid interface lead in general, to a system of simultaneous linear integral equations on the unknown source strengths. For practical purposes, an approach is utilized in which the boundaries of both the cylindrical body and the fluid are divided into a series of bands over each of which the unknown source strengths are considered to be constants. Consequently, the integral equations on the unknown source strengths give rise to a system of simultaneous linear algebraic equations on the source strengths. The coefficients appearing in these equations are in the form of definite integrals which must be evaluated numerically for a given geometry and forcing frequency.

The solution of the equations on the source strengths on the cylinder–fluid interface allows the computation of the pressure and velocity fields at any point in the infinite fluid by means of suitable integrations over the boundary sources corresponding to the acoustic fluid.

While the use of the potential approach for fluids is routine, it appears that the generalization of using three sets of sources to express the field in an elastic solid, has not been previously utilized.

To solve the coupled forced vibration problem for an arbitrary pressure distribution on the surface of the cylinder, the exciting forces are expanded in a Fourier series in θ around the circumference, such that each term can be treated separately. An illustrative numerical example is given for the axial symmetric case ($n = 0$), i.e. the case of constant distribution of excitation forces in θ, for the case of a cylinder with $L/a = 2$ and $\omega a/c = 2{\cdot}01$.

2. FORMULATION OF THE PROBLEM

An elastic cylindrical body of finite length L and radius a is submerged in an acoustic fluid of infinite extent. The cylinder is excited by surface tractions which are harmonic in time, but which may vary arbitrarily along both the axes of the cylinder and its circumference.

The exciting surface tractions are expanded into a Fourier series in θ and the pressure and velocity field components are evaluated separately for each term of the series as defined by n, the number of circumferential waves in the cylinder displacements. The problem is formulated in two steps. First, the solid cylinder and the infinite fluid with a finite cylindrical hole are considered separately. In each case, appropriate distributions of simple sources of unknown strengths are applied along the boundaries, and the stress and velocity components are derived in terms of the source strengths. Finally, the strengths of the surface source distributions are evaluated using the conditions which require the equality of tractions and normal velocities on the elastic cylinder–fluid surface interface. The pressure and velocity fields in the fluid may then be evaluated by suitable space integrations over the source distributions on the fluid surface.

2.1. *Elastic cylinder*

Consider a linearly elastic homogeneous and isotropic cylinder of length L and radius a, *in vacuo*. The displacement equations of motion are

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u} = \rho\ddot{\mathbf{u}} \tag{1}$$

and the displacement $\mathbf{u}$ is expressed in terms of its components parallel to the coordinate axes:

$$\mathbf{u} = w(r,\theta,z,t)\mathbf{k}_r+v(r,\theta,z,t)\mathbf{k}_\theta+u(r,\theta,z,t)\mathbf{k}_z. \tag{2}$$

Introducing potential functions

$$\varphi(r,\theta,z,t) = \sum_{n=0}^{\infty} \varphi_n(r,z)\cos n\theta\, e^{i\omega t} \tag{3}$$

$$\psi(r,\theta,z,t) = \sum_{n=0}^{\infty} \psi_n(r,z)\cos n\theta\, e^{i\omega t} \tag{4}$$

$$\eta(r,\theta,z,t) = \sum_{n=1}^{\infty} \eta_n(r,z)\sin n\theta\, e^{i\omega t} \tag{5}$$

the displacement components are defined as follows:

$$w(r, \theta, z, t) = \sum_{n=0}^{\infty} w_n(r, z)\cos n\theta\, e^{i\omega t} \tag{6}$$

$$v(r, \theta, z, t) = \sum_{n=1}^{\infty} v_n(r, z)\sin n\theta\, e^{i\omega t} \tag{7}$$

$$u(r, \theta, z, t) = \sum_{n=0}^{\infty} u_n(r, z)\cos n\theta\, e^{i\omega t} \tag{8}$$

where

$$w_n(r, z) = \left(\varphi_{n,r} + \frac{n}{r}\eta_n - \psi_{n,rz}\right) \tag{9}$$

$$v_n(r, z) = \left(-\frac{n}{r}\varphi_n - \eta_{n,r} + \frac{n}{r}\psi_{n,z}\right) \tag{10}$$

and

$$u_n(r, z) = \left(\varphi_{n,z} + \psi_{n,rr} + \frac{1}{r}\psi_{n,r} - \frac{n^2}{r^2}\psi_n\right). \tag{11}$$

Substituting equations (6)–(11) into equation (1), the potential functions φ_n, ψ_n and η_n satisfy the wave equations

$$\nabla_1^2\varphi_n + \left(k_1^2 - \frac{n^2}{r^2}\right)\varphi_n = 0 \tag{12}$$

$$\nabla_1^2\psi_n + \left(k_2^2 - \frac{n^2}{r^2}\right)\psi_n = 0 \tag{13}$$

$$\nabla_1^2\eta_n + \left(k_2^2 - \frac{n^2}{r^2}\right)\eta_n = 0 \tag{14}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{15a}$$

$$k_1 = \frac{\omega}{c_1} \text{ and } k_2 = \frac{\omega}{c_2}. \tag{15b}$$

The stress components may be written in terms of the potential functions:

$$\sigma_{rr} = \sum_{n=0}^{\infty} \sigma_{rr_n}(r, z)\cos n\theta\, e^{i\omega t} \tag{16}$$

$$\sigma_{r\theta} = \sum_{n=1}^{\infty} \sigma_{r\theta_n}(r, z)\sin n\theta\, e^{i\omega t} \tag{17}$$

$$\sigma_{rz} = \sum_{n=0}^{\infty} \sigma_{rz_n}(r, z)\cos n\theta\, e^{i\omega t} \tag{18}$$

$$\sigma_{\theta\theta} = \sum_{n=0}^{\infty} \sigma_{\theta\theta_n}(r, z)\cos n\theta\, e^{i\omega t} \tag{19}$$

$$\sigma_{z\theta} = \sum_{n=1}^{\infty} \sigma_{z\theta_n}(r, z)\sin n\theta\, e^{i\omega t} \tag{20}$$

$$\sigma_{zz} = \sum_{n=0}^{\infty} \sigma_{zz_n}(r, z)\cos n\theta\, e^{i\omega t} \tag{21}$$

where

$$\sigma_{rr_n} = \lambda\left(\nabla_1^2\varphi_n - \frac{n^2}{r^2}\varphi_n\right) + 2\mu\left(\varphi_{n,rr} - \frac{n}{r^2}\eta_n + \frac{n}{r}\eta_{n,r} - \psi_{n,rrz}\right) \tag{22}$$

$$\sigma_{r\theta_n} = \mu\left(\frac{2n}{r^2}\varphi_n - \frac{2n}{r}\varphi_{n,r} - \frac{2n}{r^2}\psi_{n,z} + \frac{2n}{r}\psi_{n,rz} - \eta_{n,rr} - \frac{n^2}{r^2}\eta_n + \frac{1}{r}\eta_{n,r}\right) \tag{23}$$

$$\sigma_{rz_n} = \mu\left(2\varphi_{n,rz} + \frac{2n^2}{r^3}\psi_n - \frac{n^2}{r^2}\psi_{n,r} + \psi_{n,rrr} + \frac{1}{r}\psi_{n,rr} - \psi_{n,rzz} - \frac{1}{r^2}\psi_{n,r} + \frac{n}{r}\eta_{n,z}\right) \tag{24}$$

$$\sigma_{\theta\theta_n} = \lambda\left(\nabla_1^2\varphi_n - \frac{n^2}{r^2}\varphi_n\right) + \frac{2\mu}{r}\left(\varphi_{n,r} - \frac{n^2}{r}\varphi_n + \frac{n^2}{r^2}\psi_{n,z} - \psi_{n,rz} + \frac{n}{r}\eta_n - n\eta_{n,r}\right) \tag{25}$$

$$\sigma_{z\theta_n} = \mu\left(-\frac{2n}{r}\varphi_{n,z} + \frac{n}{r}\psi_{n,zz} + \frac{n^3}{r^3}\psi_n - \frac{n}{r}\psi_{n,rr} - \frac{n}{r^2}\psi_{n,r} - \eta_{n,rz}\right) \tag{26}$$

$$\sigma_{zz_n} = \lambda\left(\nabla_1^2\varphi_n - \frac{n^2}{r^2}\varphi_n\right) + 2\mu\left(\varphi_{n,zz} + \psi_{n,rrz} - \frac{n^2}{r^2}\psi_{n,z} + \frac{1}{r}\psi_{n,rz}\right) \tag{27}$$

To formulate the problem in terms of source distributions, consider first a function $\zeta(r, \theta, z, t) = \zeta(r, \theta, z)e^{i\omega t}$ which satisfies the wave equation

$$\nabla^2\zeta + k^2\zeta = 0; \qquad k^2 = \frac{\omega^2}{c^2}. \tag{28}$$

From potential theory [1, 2], the solution of ζ at points P within the elastic cylinder and on its boundary can be first expressed in terms of a distribution of simple sources and doublets which are applied along the boundary surfaces of the finite cylinder. The value of ζ at a point P can be written

$$\zeta_P = -\iint_S \frac{e^{-ikR}}{4\pi R}\frac{\partial\varphi}{\partial n}\mathrm{d}s + \iint_S \varphi\frac{\partial}{\partial n}\left(\frac{e^{-ikR}}{R}\right)\mathrm{d}s \tag{29}$$

where the first term represents the contribution to ζ which is produced by a surface distribution of simple sources of density $-\partial\varphi/\partial n$ per unit area and the second term represents the contribution produced by a surface distribution of double sources (doublets) with axes normal to the surface and density φ per unit of area. The quantity R is the distance between the field point P at which the quantity ζ is to be evaluated and the appropriate source or doublet location, i, on the surface of the cylinder. Proceeding one step further, the function ζ can be expressed in terms of a surface distribution of simple sources only, by defining an auxiliary function ζ' in the infinite region external to the cylinder such that

$$\nabla^2\zeta' + k^2\zeta' = 0 \tag{30}$$

and ζ' and its first and second derivatives are finite in this region, and by setting the doublet strength equal to zero.

As shown by Lamb [1], (Article 290), the function ζ at the point P may be written*

$$\zeta_P = -\iint_S \frac{e^{-ikR}}{4\pi R}\left(\frac{\partial\zeta}{\partial n} + \frac{\partial\zeta'}{\partial n'}\right)ds \tag{31}$$

where ζ_P is interpreted as the function produced by a surface distribution of simple sources of strength $-[(\partial\zeta/\partial n)+(\partial\zeta'/\partial n')]$ per unit of area. The function ζ and its even normal derivatives are continuous on the cylindrical boundaries while the odd normal derivatives on the boundary of the cylinder will be discontinuous.

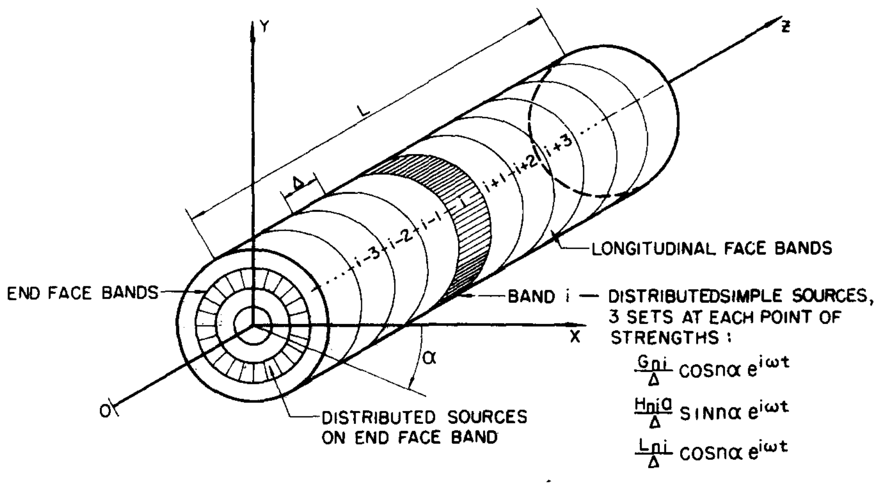

FIG. 2. Source distribution on typical band i of cylindrical boundary.

Consider therefore the solid cylinder of finite length with a distribution of simple sources on its boundaries. At each boundary point, three separate time-harmonic sources, each corresponding to one of the three elastic potentials $\varphi_n \cos n\theta$, $\psi_n \cos n\theta$ and $\eta_n \sin n\theta$, are placed. For purposes of computation, the surfaces of the cylinder are sub-divided into N bands over which the sources are distributed with strengths which vary cosinusoidally or sinusoidally in θ and harmonically in time (Fig. 2). The strengths of the sources in each band on the surface $r = a$ are considered to be constant over the coordinate z, while the strengths of the sources in each band on the surfaces $z = 0$ and $z = L$ are considered to be constant over the coordinate r. Finally, the sources are lumped at the centerline of each individual band thus giving rise to a series of circular line source distributions at locations i on the surfaces of the elastic cylinder (Fig. 3)†.

* The reader is referred to [1], p. 449 ff. for the derivation of equation (31).

† In the theory which follows, the field point P will be denoted by the subscript j and the coordinates r, θ, z, while the source point Q will be denoted by the subscript i and the coordinates $\bar{r}$, α, $\bar{z}$. See Fig. (A-1) of Appendix A.

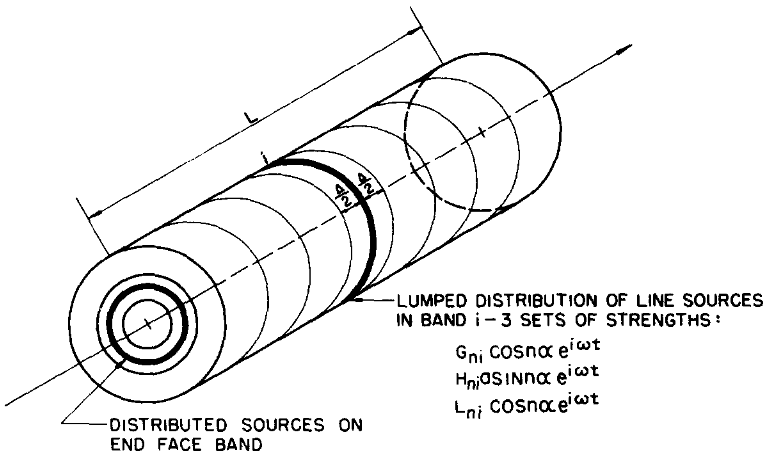

FIG. 3. Lumped source distributions on typical band i of cylindrical boundary.

Consider therefore the elastic cylinder with the three line distributions, shown in Table 1, of simple sources at each of the locations i:

TABLE 1

Elastic potential function	Strength of corresponding line sources at location i
$\varphi_n \cos n\theta e^{i\omega t}$	$G_{ni} \cos n\alpha e^{i\omega t}$
$\psi_n \cos n\theta e^{i\omega t}$	$H_{ni} a \cos n\alpha e^{i\omega t}$
$\eta_n \sin n\theta e^{i\omega t}$	$L_{ni} \sin n\alpha e^{i\omega t}$

where the complex quantities G_{ni}, H_{ni} and L_{ni} are the source strength coefficients per unit of circumferential length which are to be evaluated from the conditions at the interface between the elastic solid and the acoustic fluid.

The potential functions φ_j, ψ_j and η_j at a point j in the interior or on the boundary of the cylindrical body may be written in terms of the contributions from each of the line distributions of sources at the locations i:

$$\varphi_j = \sum_{n=0}^{\infty} \varphi_{nj} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} \varphi_{nji} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} G_{ni}\alpha_{nji} \cos n\theta\, e^{i\omega t} \tag{32}$$

$$\psi_j = \sum_{n=0}^{\infty} \psi_{nj} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} \psi_{nji} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} H_{ni}a\beta_{nji} \cos n\theta\, e^{i\omega t} \tag{33}$$

$$\eta_j = \sum_{n=1}^{\infty} \eta_{nj} = \sum_{n=1}^{\infty} \sum_{i=1}^{N} \eta_{nji} = \sum_{n=1}^{\infty} \sum_{i=1}^{N} L_{ni}\beta_{nji} \sin n\theta\, e^{i\omega t} \tag{34}$$

where the complex coefficients α_{nji} and β_{nji} are defined by the relations

$$\alpha_{nji} = -\frac{\bar{r}}{4\pi}\int_0^{2\pi} \left(\frac{e^{-ik_1R}}{R}\right) \cos n\psi\, d\psi \tag{35}$$

$$\beta_{nji} = -\frac{\bar{r}}{4\pi}\int_0^{2\pi}\left(\frac{e^{-ik_2R}}{R}\right)\cos n\psi \, d\psi \tag{36}$$

and

$$R = [(z-\bar{z})^2+\bar{r}^2+r^2-2r\bar{r}\cos\psi]^{\frac{1}{2}}. \tag{37}$$

The term φ_{nji} refers to the contribution to the potential component φ_n at the field point j (coordinates r, θ, z) from the line distribution of sources which are located at the source location i (coordinates $\bar{r}$, α, $\bar{z}$).

Substituting equations (32)–(34) into equations (16)–(27) and using the relations

$$\begin{aligned}
\nabla_1^2\varphi_{ni} &= \left(-k_1^2+\frac{n^2}{r^2}\right)\varphi_n \\
\nabla_1^2\psi_{ni} &= \left(-k_2^2+\frac{n^2}{r^2}\right)\psi_n \\
\nabla_1^2\eta_{ni} &= \left(-k_2^2+\frac{n^2}{r^2}\right)\eta_n
\end{aligned} \tag{38}$$

the stresses σ at a field point j are written in terms of the source strengths G_{ni}, H_{ni} and L_{ni}:*

$$\sigma_{rr,j} = \sum_{n=0}^{\infty}\sigma_{rr,nj}(r,z)\cos n\theta e^{i\omega t}$$

where

$$\sigma_{rr,nj} = \sum_{i=1}^{N}\left\{G_{ni}\left(-\frac{\lambda\omega^2}{c_1^2}\alpha_{nji}+2\mu\alpha_{nji}^{rr}\right)+L_{ni}2\mu\left(-\frac{n}{r^2}\beta_{nji}+\frac{n}{r}\beta_{nji}^{r}\right)-aH_{ni}2\mu\beta_{nji}^{rrz}\right\} \tag{39}$$

$$\sigma_{r\theta,j} = \sum_{n=1}^{\infty}\sigma_{r\theta,nj}(r,z)\sin n\theta e^{i\omega t}$$

where

$$\begin{aligned}
\sigma_{r\theta,nj} = \mu\sum_{i=1}^{N}\Bigg\{&G_{ni}\left(\frac{2n}{r^2}\alpha_{nji}-\frac{2n}{r}\alpha_{nji}^{r}\right)+H_{ni}a\left(-\frac{2n}{r^2}\beta_{nji}^{z}+\frac{2n}{r}\beta_{nji}^{rz}\right) \\
&+L_{ni}\left(-\beta_{nji}^{rr}-\frac{n^2}{r^2}\beta_{nji}+\frac{1}{r}\beta_{nji}^{r}\right)\Bigg\}
\end{aligned} \tag{40}$$

$$\sigma_{rz,j} = \sum_{n=0}^{\infty}\sigma_{rz,nj}(r,z)\cos n\theta e^{i\omega t}$$

where

$$\sigma_{rz,nj} = \mu\sum_{i=1}^{N}\left\{G_{ni}\left(2\alpha_{nji}^{rz}\right)+H_{ni}a\left(-2\beta_{nji}^{rzz}-\frac{\omega^2}{c_2^2}\beta_{nji}^{r}\right)+L_{ni}\left(\frac{n}{r}\beta_{nji}^{z}\right)\right\} \tag{41}$$

$$\sigma_{\theta\theta,j} = \sum_{n=0}^{\infty}\sigma_{\theta\theta,nj}(r,z)\cos n\theta e^{i\omega t}$$

* The superscripts on the quantities α_{nji} and β_{nji} represent differentiation of the integrals of equations (35)–(36), e.g. $\alpha_{nji}^{rz} = \partial^2\alpha_{nji}/\partial r\,\partial z$ etc.

where

$$\sigma_{\theta\theta,nj} = \sum_{i=1}^{N} \left\{ G_{ni} \left[-\frac{\lambda\omega^2}{c_1^2}\alpha_{nji} + \frac{2\mu}{r}\left(\alpha^r_{nji} - \frac{n^2}{r}\alpha_{nji}\right)\right] + H_{ni}a\left[\frac{2\mu}{r}\left(\beta^z_{nji} - \beta^{rz}_{nji}\right)\right] + L_{ni}\left[\frac{2\mu}{r}\left(\frac{n}{r}\beta_{nji} - n\beta^r_{nji}\right)\right]\right\} \tag{42}$$

$$\sigma_{z\theta,j} = \sum_{n=1}^{\infty} \sigma_{z\theta,nj} \sin n\theta e^{i\omega t}$$

where

$$\sigma_{z\theta,nj} = \mu \sum_{i=1}^{N} \left\{ G_{ni}\left(-\frac{2n}{r}\alpha^z_{nji}\right) + H_{ni}a\left(\frac{2n}{r}\beta^{zz}_{nji} + \frac{n\omega^2}{rc_2^2}\beta_{nji}\right) + L_{ni}\left(-\beta^{rz}_{nji}\right)\right\} \tag{43}$$

and

$$\sigma_{zz,j} = \sum_{n=0}^{\infty} \sigma_{zz,nj}(r, z) \cos n\theta e^{i\omega t}$$

where

$$\sigma_{zz,nj} = \sum_{i=1}^{N} \left\{ G_{ni}\left(-\frac{\lambda\omega^2}{c_1^2}\alpha_{nji} + 2\mu\alpha^{zz}_{nji}\right) + aH_{ni}2\mu\left(-\beta^{zzz}_{nji} - \frac{\omega^2}{c_2^2}\beta^z_{nji}\right)\right\} \tag{44}$$

Substituting equations (32)–(34) into equations (6)–(11), and differentiating with respect to time, the velocity components at the field point j become

$$\dot{w}_j = \sum_{n=0}^{\infty} \dot{w}_{n,j}(r, z) \cos n\theta e^{i\omega t} \tag{45}$$

$$\dot{v}_j = \sum_{n=1}^{\infty} \dot{v}_{n,j}(r, z) \sin n\theta e^{i\omega t} \tag{46}$$

$$\dot{u}_j = \sum_{n=0}^{\infty} \dot{u}_{n,j}(r, z) \cos n\theta e^{i\omega t} \tag{47}$$

where

$$\dot{w}_{n,j} = i\omega \sum_{i=1}^{N} \left\{ G_{ni}(\alpha^r_{nji}) + H_{ni}a(-\beta^{rz}_{nji}) + L_{ni}\left(\frac{n}{r}\beta_{nji}\right)\right\} \tag{48}$$

$$\dot{v}_{n,j} = i\omega \sum_{i=1}^{N} \left\{ G_{ni}\left(\frac{n}{r}\alpha_{nji}\right) + H_{ni}a\left(-\frac{n}{r}\beta^z_{nji}\right) + L_{ni}(\beta^r_{nji})\right\} \tag{49}$$

and

$$\dot{u}_{n,j} = i\omega \sum_{i=1}^{N} \left\{ G_{ni}(\alpha^z_{nji}) + H_{ni}a\left(-\frac{\omega^2}{c_2^2}\beta_{nji} - \beta^{zz}_{nji}\right)\right\}. \tag{50}$$

The complex coefficients α_{nji} and their derivatives are defined by equation (35) and equations (51)–(58) which follow:

$$\alpha^r_{nji} = \frac{\bar{r}}{4\pi}\int_0^{2\pi} \left(\frac{e^{-ik_1R}}{R^3}\right)(1 + ik_1R)(r - \bar{r}\cos\psi)\cos n\psi \, d\psi \tag{51}$$

$$\alpha_{nji}^{\bar{z}} = -\frac{\bar{r}}{4\pi}\int_0^{2\pi}\left(\frac{e^{-ik_1R}}{R^3}\right)(1+ik_1R)(\bar{z}-z)\cos n\psi\,\mathrm{d}\psi \tag{52}$$

$$\alpha_{nji}^{zz} = \frac{\bar{r}}{4\pi}\int_0^{2\pi}\left(\frac{e^{-ik_1R}}{R^3}\right)\left[(\bar{z}-z)^2\left(-\frac{3(1+ik_1R)+k_1^2R^2}{R^2}\right)+(1+ikR)\right]\cos n\psi\,\mathrm{d}\psi \tag{53}$$

$$\alpha_{nji}^{zr} = -\frac{\bar{r}}{4\pi}(\bar{z}-z)\int_0^{2\pi}\left(\frac{e^{-ik_1R}}{R^5}\right)[-3(1+ik_1R)+k_1^2R^2](r-\bar{r}\cos\psi)\cos n\psi\,\mathrm{d}\psi \tag{54}$$

$$\begin{aligned}\alpha_{nji}^{zzz} = -\frac{\bar{r}(\bar{z}-z)}{4\pi}\int_0^{2\pi}\Bigg\{&\left(\frac{e^{-ik_1R}}{R^7}\right)[15(1+ik_1R)-6k^2R^2-ik_1^3R^3](\bar{z}-z)^2\\&+\left(\frac{e^{-ik_1R}}{R^5}\right)[-9(1+ik_1R)+3k_1^2R^2]\Bigg\}\cos n\psi\,\mathrm{d}\psi\end{aligned} \tag{55}$$

$$\begin{aligned}\alpha_{nji}^{rr} = \frac{\bar{r}}{4\pi}\int_0^{2\pi}\Bigg\{&\left(\frac{e^{-ik_1R}}{R^5}\right)[-3(1+ik_1R)+k_1^2R^2](r-\bar{r}\cos\psi)^2\\&+\left(\frac{e^{-ik_1R}}{R^3}\right)(1+ik_1R)\Bigg\}\cos n\psi\,\mathrm{d}\psi\end{aligned} \tag{56}$$

$$\begin{aligned}\alpha_{nji}^{zrr} = -\frac{\bar{r}}{4\pi}\int_0^{2\pi}\Bigg\{&\left(\frac{e^{-ik_1R}}{R^5}\right)[-3(1+ik_1R)+k_1^2R^2]+\left(\frac{e^{-ik_1R}}{R^7}\right)(r-\bar{r}\cos\psi)^2\\&\times[15(1+ik_1R)-6k_1^2R^2-ik_1^3R^3]\Bigg\}(\bar{z}-z)\cos n\psi\,\mathrm{d}\psi\end{aligned} \tag{57}$$

$$\begin{aligned}\alpha_{nji}^{rzz} = \frac{\bar{r}}{4\pi}\int_0^{2\pi}(r-\bar{r}\cos\psi)\Bigg\{&\left(\frac{e^{-ik_1R}}{R^7}\right)[15(1+ik_1R)-6k_1^2R^2-ik_1^3R^3](\bar{z}-z)^2\\&+\left(\frac{e^{-ik_1R}}{R^5}\right)[-3(1+ik_1R)+k_1^2R^2]\Bigg\}\cos n\psi\,\mathrm{d}\psi\end{aligned} \tag{58}$$

The corresponding derivatives of the complex coefficients β_{nji} are evaluated by replacing $k_1 = \omega/c_1$ by $k_2 = \omega/c_2$ in equations (51)–(58).

2.2. *Acoustic fluid*

Consider the routine case of an acoustic fluid of infinite extent with a cylindrical cavity of radius a and length L. The motions of the fluid are governed by a velocity potential $\Phi(r, \theta, z, t)$,

$$\Phi(r, \theta, z, t) = \sum_{n=0}^{\infty} \Phi_n(r, z)\cos n\theta e^{i\omega t} \tag{59}$$

which satisfies the wave equation,

$$\nabla_1^2\Phi_n+\left(k_3^2-\frac{n^2}{r^2}\right)\Phi_n = 0 \tag{60}$$

where

$$k_3 = \frac{\omega}{c}. \tag{61}$$

The velocity and pressure components at points in the medium are given by the relations:

$$v_r = -\frac{\partial \Phi}{\partial r} = -\sum_{n=0}^{\infty} \frac{\partial \Phi_n(r, z)}{\partial r} \cos n\theta e^{i\omega t} \tag{62}$$

$$v_\theta = -\frac{\partial \Phi}{r\,\partial \theta} = \sum_{n=1}^{\infty} \frac{n\Phi_n(r, z)}{r} \sin n\theta e^{i\omega t} \tag{63}$$

$$v_z = -\frac{\partial \Phi}{\partial z} = -\sum_{n=0}^{\infty} \frac{\partial \Phi_n(r, z)}{\partial z} \cos n\theta e^{i\omega t} \tag{64}$$

$$p = \rho_f \frac{\partial \Phi}{\partial t} = \sum_{n=0}^{\infty} \rho_f i\omega \Phi_n(r, z) \cos n\theta e^{i\omega t} \tag{65}$$

Proceeding exactly as in the case of the finite elastic cylinder, the problem is formulated in terms of a distribution of simple sources on the internal boundaries of the infinite fluid, $z = 0$, L and $r = a$.* At each location i, a line distribution of simple sources of strengths $C_{ni} \cos n\alpha e^{i\omega t}$, corresponding to the potential function $\Phi_n \cos n\theta e^{i\omega t}$, is placed. The source locations i are the same as those which were used in the case of the finite elastic cylinder.

The potential function Φ_j at a point j in the interior or on the internal boundary of the fluid can be written in terms of the contributions from each of the line distributions of sources at the locations i,

$$\Phi_j = \sum_{n=0}^{\infty} \Phi_{nj} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} \Phi_{nji} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} C_{ni}\gamma_{nji} \cos n\theta e^{i\omega t}. \tag{66}$$

The complex coefficient γ_{nji} is given by the relation

$$\gamma_{nji} = -\frac{\bar{r}}{4\pi} \int^{2\pi} \left[\frac{e^{-ik_3 R}}{R} \right] \cos n\psi \, d\psi. \tag{67}$$

Substituting equation (66) into equations (62)–(65), the pressure and velocities at a field point j in the fluid are written in terms of the source strengths C_{ni}:

On the surface $r = a$

$$\sigma_{rr,j} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} \rho_f \omega i C_{ni} \gamma_{nji} \cos n\theta e^{i\omega t} \tag{68}$$

On the surface $z = 0$ and $z = L$

$$\sigma_{zz,j} = \sum_{n=0}^{\infty} \sum_{i=1}^{N} \rho_f \omega i C_{ni} \gamma_{nji} \cos n\theta e^{i\omega t} \tag{69}$$

$$\dot{w}_j = -\sum_{n=0}^{\infty} \sum_{i=1}^{N} C_{ni} \gamma^r_{nji} \cos n\theta e^{i\omega t} \tag{70}$$

* This can be done provided Φ approaches a form Ce^{-ikR}/R as $R \to \infty$, i.e. that there are no sources of sound at infinity.

$$\dot{v}_j = \sum_{n=1}^{\infty} \sum_{i=1}^{N} C_{ni} \frac{n}{r} \gamma_{nji} \sin n\theta e^{i\omega t} \tag{71}$$

$$\dot{u}_j = - \sum_{n=0}^{\infty} \sum_{i=1}^{N} C_{ni} \gamma^z_{nji} \cos n\theta e^{i\omega t} \tag{72}$$

The complex coefficients γ^r_{nji} and γ^z_{nji} are obtained by substituting $k_3 = \omega/c$ for $k_1 = \omega/c_1$ in equations (51) and (52) respectively.

2.3. *Relations on the interface between the fluid and the elastic cylinder*

Let the cylindrical shell be excited by the time-harmonic boundary tractions

$$\sigma_{rr} = \sum_{n=0}^{\infty} P_{rr,n}(r, z) \cos n\theta e^{i\omega t} \qquad \text{on } r = a \tag{73}$$

$$\sigma_{zz} = \sum_{n=0}^{\infty} P_{zz,n}(r, z) \cos n\theta e^{i\omega t} \qquad \text{on } z = 0 \text{ and } z = L \tag{74}$$

as shown in Fig. 4.

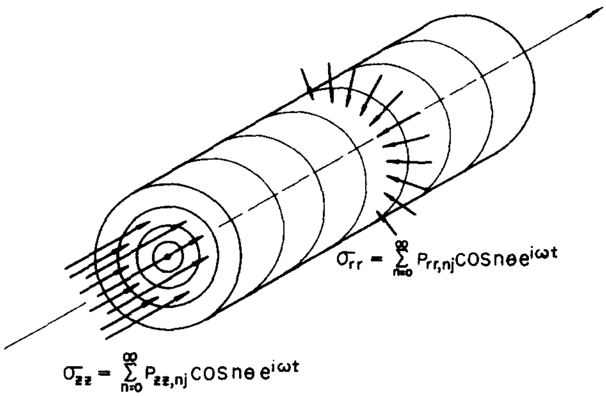

FIG. 4. Applied tractions on typical bands.

The equations stating the equality of the tractions and velocities at the fluid–cylinder interfaces can be written at each field point j on the surfaces.* Defining the coefficients $P_{rr,nj}$ and $P_{zz,nj}$ as the expansion coefficients of the externally applied normal tractions at the field points j on the indicated boundaries, the conditions on the tractions and the normal velocities are given below at each point j:

Point j on the surface $r = a$

$$\sigma_{rr,nj\,\text{shell}} = \sigma_{rr,nj\,\text{fluid}} + P_{rr,nj} \tag{75}$$

$$\sigma_{rz,nj\,\text{shell}} = 0 \tag{76}$$

$$\sigma_{r\theta,nj\,\text{shell}} = 0 \tag{77}$$

$$\dot{w}_{n,j\,\text{shell}} = \dot{w}_{n,j\,\text{fluid}} \tag{78}$$

* The points j are taken at the same locations i as the line sources.

Point j on the surfaces z = 0 and z = L

$$\sigma_{zz,nj\,\mathrm{shell}} = \sigma_{zz,nj\,\mathrm{fluid}} + P_{zz,nj} \tag{79}$$

$$\sigma_{rz,nj\,\mathrm{shell}} = 0 \tag{80}$$

$$\sigma_{z\theta,nj\,\mathrm{shell}} = 0 \tag{81}$$

$$\dot{u}_{n,j\,\mathrm{shell}} = \dot{u}_{n,j\,\mathrm{fluid}}. \tag{82}$$

The stress components σ_{rz}, $\sigma_{r\theta}$, σ_{zr} and $\sigma_{z\theta}$ are set equal to zero in the above equations since the fluid is assumed to be inviscid and hence not capable of sustaining shear stresses. Substituting the appropriate expressions from equations (39)–(50), equations (68)–(72) and equations (73) and (74) into equations (79)–(82), the equations may be written in terms of the source strengths G_{ni}, L_{ni}, H_{ni} and C_{ni}:

On the surface r = a

$$\sum_{i=1}^{N}\left\{G_{ni}\left(-\frac{\lambda\omega^2}{c_1^2}\alpha_{nji}+2\mu\alpha_{nji}^{rr}\right)+L_{ni}2\mu\left(-\frac{n}{r^2}\beta_{nji}+\frac{n}{r}\beta_{nji}^{r}\right)-H_{ni}a\beta_{nji}^{rrz}2\mu\right\}_{r=a}$$
$$=\sum_{i=1}^{N}\rho_f\omega i C_{ni}\gamma_{nji}+P_{rr,nj} \tag{83}$$

$$\sum_{i=1}^{N}\left\{G_{ni}(2\alpha_{nji}^{rz})+H_{ni}a\left(-2\beta_{nji}^{rzz}-\frac{\omega^2}{c_2^2}\beta_{nji}^{r}\right)+L_{ni}\left(\frac{n}{r}\beta_{nji}^{z}\right)\right\}_{r=a}=0 \tag{84}$$

$$\sum_{i=1}^{N}\left\{G_{ni}\left(\frac{2n}{r^2}\alpha_{nji}-\frac{2n}{r}\alpha_{nji}^{r}\right)+H_{ni}a\left(-\frac{2n}{r^2}\beta_{nji}^{z}+\frac{2n}{r}\beta_{nji}^{rz}\right)\right.$$
$$\left.+L_{ni}\left(-\beta_{nji}^{rr}-\frac{n^2}{r^2}\beta_{nji}+\frac{1}{r}\beta_{nji}^{r}\right)\right\}_{r=a}=0 \tag{85}$$

$$\sum_{i=1}^{N} i\omega\left\{G_{ni}\alpha_{nji}^{r}+H_{ni}a(-\beta_{nji}^{rz})+L_{ni}\left(\frac{n}{r}\beta_{nji}\right)\right\}_{r=a}=-\sum_{i=1}^{N}C_{ni}\gamma_{nji}^{r} \tag{86}$$

On the surfaces z = 0 and z = L

$$\sum_{i=1}^{N}\left\{G_{ni}\left(-\frac{\lambda\omega^2}{c_1^2}\alpha_{nji}+2\mu\alpha_{nji}^{zz}\right)+aH_{ni}2\mu\left(-\beta_{nji}^{zzz}-\frac{\omega^2}{c_2^2}\beta_{nji}^{z}\right)\right\}$$
$$=\sum_{i=1}^{N}\rho_f\omega i C_{ni}\gamma_{nji}+P_{zz,nj} \tag{87}$$

$$\sum_{i=1}^{N}\left\{G_{ni}(2\alpha_{nji}^{rz})+H_{ni}a\left(-2\beta_{nji}^{rzz}-\frac{\omega^2}{c_2^2}\beta_{nji}^{r}\right)+L_{ni}\left(\frac{n}{r}\beta_{nji}^{z}\right)\right\}=0 \tag{88}$$

$$\sum_{i=1}^{N}\left\{G_{ni}\left(-\frac{2n}{r}\alpha_{nji}^{z}\right)+H_{ni}a\left(\frac{2n}{r}\beta_{nji}^{zz}+\frac{n\omega^2}{rc_2^2}\beta_{nji}\right)+L_{ni}(-\beta_{nji}^{rz})\right\}=0 \tag{89}$$

$$\sum_{i=1}^{N} i\omega\left\{G_{ni}(\alpha_{nji}^{z})+H_{ni}a\left(-\frac{\omega^2}{c_2^2}\beta_{nji}-\beta_{nji}^{zz}\right)\right\}=-\sum_{i=1}^{N}C_{ni}\gamma_{nji}^{z}. \tag{90}$$

The set of simultaneous non-homogeneous linear algebraic equations, equations (83)–(90) may be solved for the complex source strength coefficients G_{ni}, L_{ni}, H_{ni} and

C_{ni} at each source location i. A brief description of the solution of these equations for both the real and imaginary parts of these coefficients is given in Section 4.

Once the fluid source coefficients C_{ni} are known, the pressure and velocity fields in the fluid can be evaluated by direct integration over the fluid sources Φ.

2.4. *Evaluation of the fluid pressure and velocity fields*

The component of the fluid potential corresponding to the integer n, at a field point j in the infinite fluid medium is expressed in terms of the boundary sources on the fluid surfaces $r = a$, $z = 0$ and $z = L$ by the relation

$$\Phi_{nj} = \sum_{i=1}^{N} C_{ni}\gamma_{nji} \cos n\theta e^{i\omega t}. \tag{91}$$

The complex coefficients γ_{nji} are given by equation (67); the quantity R is given by equation (37) and the values of the complex coefficients C_{ni} are obtained from the solution of the system of simultaneous equations, equations (83)–(90). Substituting equation (91) into equation (65), the pressure component $p_n(r, \theta, z, t)$ at a point j in the medium is

$$\frac{p_{n,j}}{\rho_f} = \sum_{i=1}^{N} i\omega C_{ni}\gamma_{nji} \cos n\theta e^{i\omega t}. \tag{92}$$

The velocity field components $\dot{w}_{n,j}$, $\dot{v}_{n,j}$ and $\dot{u}_{n,j}$ may be evaluated from equations (70)–(72) respectively.

It should be noted that the pressure and velocity components which are found from equation (92) and equations (70)–(72) are complex quantities. The magnitude and phase of these quantities may then be computed as shown in Section 4. A considerably simplified asymptotic expression for the pressure $p_{n,j}$ in the fluid at large distances R from the cylindrical body, i.e. the far field pressures in the fluid, can be derived in terms of the fluid source strength coefficients C_{ni} and a prescribed distance and slope angle, R_0 and $\bar{\zeta}$ respectively, as shown in Fig. 5.

$$\left(\frac{p_{n,j}}{\rho_f}\right)_{\text{far field}} = -\frac{i^{n+1}\omega \cos n\theta \exp[i\omega(t-R_0/c)]}{2R_0} \sum_{i=1}^{N} C_{ni}\bar{r} \exp(ik_3\bar{z} \sin \bar{\zeta})J_n(k_3\bar{r} \cos \bar{\zeta}). \tag{93}$$

It should be noted that once $p_{n,j}$ has been evaluated from equation (93) for a particular point P_1 (with a specified value of R_0 and $\bar{\zeta}$), it can easily be evaluated for any point P_j which lies on the line of R_0, by changing the scale factor

$$\frac{\exp(i\omega R_0/c)}{R_0}$$

in the equation for the pressure. The use of equation (93) therefore greatly simplifies the numerical computation of the far field fluid pressure since it eliminates the numerical computation of the γ_{nji} integral coefficients for each point j in the fluid.

3. AXI-SYMMETRIC CASE, $n = 0$

For the axi-symmetrical case, $n = 0$, the θ dependence of the potential functions vanishes.

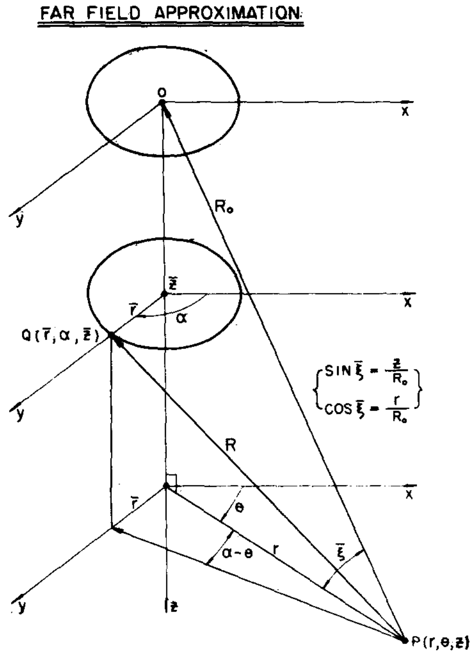

FIG. 5. Far field fluid pressure evaluation.

The displacement components at points in the elastic cylinder can be expressed in terms of two potential functions only

$$\Phi(r, z, t) = \Phi_0(r, z)e^{i\omega t} \tag{94}$$

$$\psi(r, z, t) = \psi_0(r, z)e^{i\omega t} \tag{95}$$

which satisfy the wave equations, (12) and (13), with $n = 0$.

Proceeding exactly as in the general case, the problem is formulated in terms of two separate line distributions of simple sources on the surface $r = a$ and $z = 0$ and L, of the elastic cylinder.

The potential functions φ_{0j} and ψ_{0j} at a point j on the interior or on the boundary of the cylindrical body are written in terms of the contributions from each of the two line distributions of sources at the locations i:

$$\Phi_{0j} = \sum_{i=1}^{N} G_{0i}\alpha_{0ji}e^{i\omega t} \tag{96}$$

$$\psi_{0j} = \sum_{i=1}^{N} H_{0i}a\beta_{0ji}e^{i\omega t} \tag{97}$$

where

$$\alpha_{0ji} = -\frac{\bar{r}}{4\pi}\int_0^{2\pi}\left(\frac{e^{-ik_1R}}{R}\right)d\psi \tag{98}$$

$$\beta_{0ji} = -\frac{\bar{r}}{4\pi}\int_0^{2\pi}\left(\frac{e^{-ik_2R}}{R}\right)d\psi \tag{99}$$

and R is given by equation (37).

The stress and velocity components at each field point j are written in terms of the complex source strengths G_{0i} and H_{0i} by setting $n = 0$ in equations (39)–(50), respectively, and noting that $L_{0i} \equiv 0$.

For the acoustic fluid, the problem is formulated, as in the general case, in terms of line distributions of simple sources on the surface $r = a$ and $z = 0$ and L, of the fluid. A single line distribution of simple sources of strength $C_{0i}e^{i\omega t}$, corresponding to the elastic potential function $\Phi_0 e^{i\omega t}$ is required at each location i. Hence,

$$\Phi_{0j} = \sum_{i=1}^{N} C_{0i}\gamma_{0ji}e^{i\omega t} \tag{100}$$

where the complex coefficient γ_{0ji} is obtained from equation (98) by substituting k_3 for k_1. The stress and velocity components at each field point j are obtained by setting $n = 0$ in equations (68)–(72), respectively.

The conditions on the source strengths G_{0i}, H_{0i} and C_{0i} on the cylinder-fluid interface are given by equations (83)–(90) in which n is set equal to zero and $L_{ni} \equiv 0$. This set of simultaneous nonhomogeneous linear algebraic equations may be solved for the coefficients G_{0i}, H_{0i} and C_{0i} at each source location i on the interface. Once the fluid source coefficients C_{0i} are known, the pressure and velocity fields in the fluid are obtained by setting $n = 0$ in equation (92) and equations (70)–(72), respectively.

The asymptotic expression for the far field pressure $p_{0,j}$ is obtained by setting $n = 0$ in equation (93), i.e.

$$\left(\frac{p_{0,j}}{\rho_f}\right)_{\text{far field}} = -\frac{i\omega\exp[i\omega(t-R_0/c)]}{2R_0}\sum_{i=1}^{N} C_{0i}\bar{r}\exp(ik_3\bar{z}\sin\zeta)J_0(k_3\bar{r}\cos\zeta). \tag{101}$$

4. COMPUTATIONAL PROCEDURES

The computational effort required for the application of this method to problems of practical interest is considerable, and requires the use of high speed electronic computing equipment. The computations are conveniently divided into three major parts:

(a) Evaluation of the coefficients α_{nji}, β_{nji} and γ_{nji} and their space derivatives, using equations (35), (36) and equations (51)–(58).

(b) Solution of equations, equations (83)–(90) for the source strength coefficients G_{ni}, H_{ni}, L_{ni} and C_{ni}.

(c) Evaluation of the pressure and velocity fields at desired points in the infinite fluid, equations (92), (93) and equations (70)–(72).

The major problems are concerned with the computations required for Case (b) in which large systems of linear simultaneous algebraic equations on the source strength coefficients must be solved. For practical purposes, it is convenient to evaluate both the complex coefficients α_{nji}, β_{nji}, γ_{nji} and the complex source strengths G_{ni}, H_{ni}, L_{ni} and C_{ni}

in terms of their real and imaginary parts. Consequently, if the cylinder–fluid interface is divided into N bands, sets of $8N$ ($n \neq 0$) and $6N$ ($n = 0$) simultaneous equations are obtained on the real and imaginary parts of the source strength coefficients. In each case, however, the computations can be reduced to the evaluation of a number of systems of $2N$ simultaneous equations.

5. NUMERICAL EXAMPLE—AXI-SYMMETRICAL CASE $n = 0$

For illustrative purposes, the following axi-symmetrical problem is solved. An elastic cylinder of radius a and length L, immersed in an infinite acoustic fluid, undergoes an electromagnetically induced time-harmonic uniform strain, $\varepsilon_{zz} = \varepsilon_0 e^{i\omega t}$, in the axial direction, while the radial and circumferential strains ε_{rr} and $\varepsilon_{\theta\theta}$ are kept equal to zero (Fig. 6(a)). By superposition, the pressure field that is produced in the fluid by the straining of the cylinder will be equivalent to the fluid pressures produced by a set of fictitious surface tractions which are applied to the solid cylinder in the ratio

$$\frac{\sigma_{rr}}{\sigma_{zz}} = \frac{\lambda}{\lambda + 2\mu} = \frac{\nu}{1-\nu}. \tag{102}$$

To illustrate this, consider first, the cylinder under the action of applied surface tractions σ_{rr} and σ_{zz} (Fig. 6(b)). These tractions are chosen so as to bring the cylinder

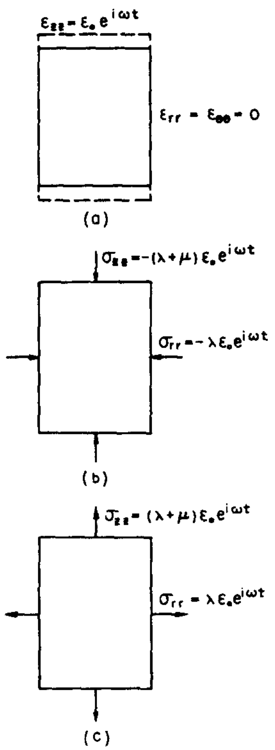

FIG. 6. Illustrative problem.

back to its original unstrained state, i.e.

$$\begin{aligned}\sigma_{rr} &= -\lambda\varepsilon_0 e^{i\omega t}\\ \sigma_{zz} &= -(\lambda+2\mu)\varepsilon_0 e^{i\omega t}.\end{aligned} \tag{103}$$

Finally, a set of surface tractions which are equal and opposite to those of equations (103) are applied to the cylinder (Fig. 6(c)).

The superposition of the three states of stress shown in Fig. 6 indicates that the pressure field which is produced in the fluid by the uniform straining of the cylinder, $\varepsilon_{zz} = \varepsilon_0 e^{i\omega t}$, is equivalent to that which is produced by the fictitious surface tractions of Fig. 6(c), namely

$$\begin{aligned}\sigma_{rr} &= \lambda\varepsilon_0 e^{i\omega t}\\ \sigma_{zz} &= (\lambda+2\mu)\varepsilon_0 e^{i\omega t}.\end{aligned} \tag{104}$$

The following parameters are used in the numerical example:

(1) *Elastic cylinder*
$L = 2a$
$\nu = \frac{1}{4}$(i.e. $\lambda = \mu$)
$\mu = 12 \times 10^6$ lb/in^2
$w = 0{\cdot}2833$ lb/in^3

(2) *Acoustic fluid*
$w_f = 62{\cdot}5$ lb/ft^3
$c_3 = 5000$ ft/sec

(3) *Symmetrical loading*
$\lambda\varepsilon_0 = 10^3$ lb/in^2
$P_{rr,0j} = 10^3 e^{i\omega t}$ — j on the surface $r = a$
$P_{zz,0j} = 3 \times 10^3 e^{i\omega t}$ — j on surface $z = 0$ and $z = L$

With the input values chosen, the propagation velocities of the pressure and shear waves in the elastic body become respectively

$$c_1 = \frac{\lambda + 2\mu}{\rho} = \frac{3\mu g}{w} = 18{,}460 \text{ ft/sec}$$

$$c_2 = \frac{\mu}{\rho} = \frac{\mu g}{w} = 10{,}660 \text{ ft/sec}$$

$$c_3(\text{fluid}) = 5000 \text{ ft/sec}.$$

The following values of the nondimensional parameters $\bar{k}_n = \omega a/c_n$ were used in the computations:

$$\bar{k}_1 = 0{\cdot}5444$$

$$\bar{k}_2 = 0{\cdot}9425$$

$$\bar{k}_3 = 2{\cdot}0100$$

For example, a possible combination of forcing frequency ω and cylinder radius for which the numerical results would apply is $\omega = 400$ c/s and $a = 4$ ft.

Numerical computations for the absolute value of the pressure $p_{0,j}$ have been carried out along rays ranging from 0°(22½°)90° as shown in Figs. 7 and 8. In each figure, the lines of constant pressure in the fluid are shown.

The results which are plotted have been derived using equation (92) and checked beyond $R_0 = 20a$ by the asymptotic pressure formula, equation (101).

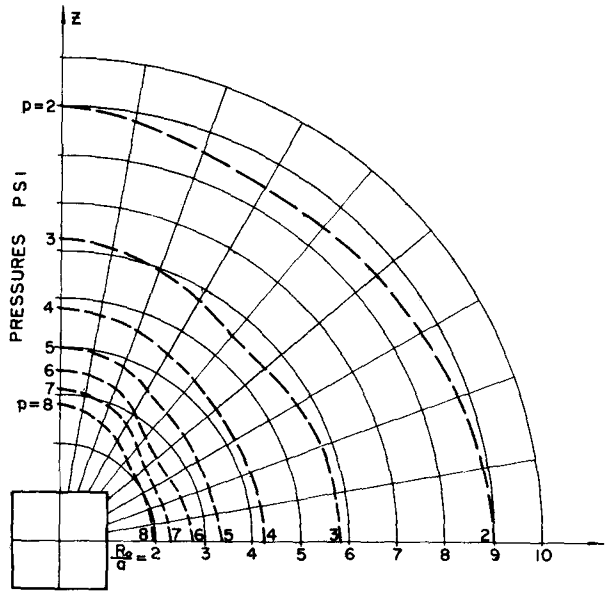

FIG. 7. Fluid pressure; $R_0/a < 10$.

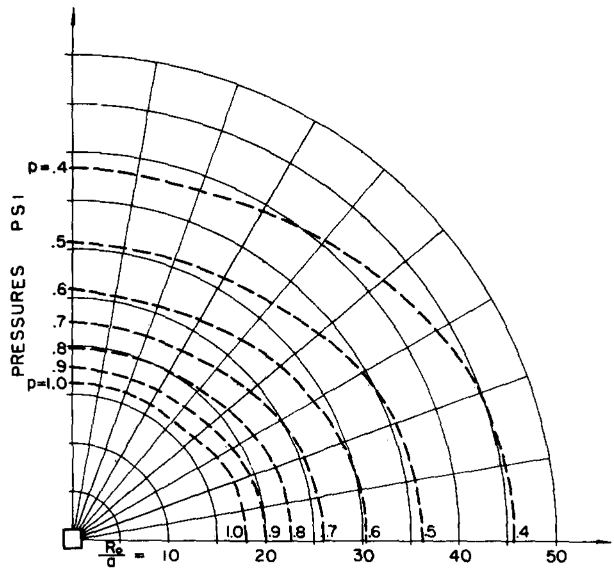

FIG. 8. Fluid pressures; $R_0/a > 10$.

6. CONCLUSIONS

A method for the evaluation of pressure and velocity fields in an infinite acoustic fluid, due to the harmonic excitation of an elastic circular cylindrical body of finite length has been presented in the preceding sections. The excitation of the cylinder, while harmonic in time, may be arbitrary in both θ and z. For arbitrary variations of surface tractions, the tractions may be expanded into Fourier series in θ and the response evaluated for each component n of the series.

The present theory may find useful applications in evaluating the pressure fields in a fluid due to the harmonic excitation of large transducers. In such transducers, changes of strain are produced electromagnetically. This situation can be treated generally by the introduction of equal and opposite fictitious tractions at the cylinder–fluid interface. These forces are selected such that one set just cancels out the electromagnetically induced strains; the pressure field produced by the other opposite set of forces is then determined by the methods of the present paper. To illustrate this approach, a numerical example for the case of an axisymmetrical excitation ($n = 0$) has been solved in Section 5 for a steel cylinder in water.

A second possible area of applications for the present theory is on problems involving the evaluation of the fluid pressures produced by the harmonic excitations of submerged thick cylindrical shells of finite length. The extension of this work to more complex geometries is also under way.

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Zusammenfassung—Eine Methode wird ausgeführt zur Bestimmung des Flüssigkeitsdruckes und des Geschwindigkeitsfeldes das durch erzwungene Schwingungen eines elastischen, in ein akustisches Medium eingetauchten Kreiszylinders endlicher Länge hervorgerufen wird. Das vorgeschlagene Verfahren macht von der Potentialtheorie Gebrauch bei dem drei Potentiale des elastischen Körpers und das eine Potential der Flüssigkeit als eine Verteilung von vier Quellengruppen gedeutet werden, die mit unbekannter Stärke an der Berührungsfläche des festen und flüssigen Körpers wirken. Für gegebene Erregung liefern die Verträglichkeitsbedingungen für Spannungen und Geschwindigkeiten an der Berührungsfläche die gesuchten Stärken und gestatten die Bestimmung des Druck- und Geschwindigkeitsfeldes in der Flüssigkeit.

Абстракт—Представлен метод определения полей давления жидкости и скорости, произведенных вынужденными колебаниями эластичного круглого цилиндрического тела определенной длины, погруженного в акустическую жидкость. Употребляется подход со стороны теории потенциалов в которой три потенциала связанные с эластичным телом и один потенциал связанный с акустической жидкостью истолковываются в виде распределения четырех групп источников неизвестной силы на поверхности раздела жидкости и эластичного тела. Для данного возбуждения, условия совместимости на нагрузках и скорости на поверхности раздела жидкости и эластичного тела приводят к последующему определению полей давления и скорости, имеющих место в жидкости.